Synopsis for PhD programme

Title of the research**:**

ON ANALYTICAL AND NUMERICAL SOLUTIONS OF SOME NONLINEAR DIFFERENTIAL EQUATIONS

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In many branches of applied mathematics, physical, biological and engineering sciences, evolution of dynamical systems are found to be described by system of coupled nonlinear ordinary or partial differential equations (ODEs/PDEs). If the system possess highly nonlinear effects or the number of variables is large enough, the motion of the system become complex in the sense that it depends sensitively on the choice of the initial conditions or on the parameters (appear as the influence of the environment or surroundings of the sub-system under consideration) involve in the equations. In the process of study of dynamical system, one first verifies whether the system is integrable. If not, the most successful technique is i) to eliminate nonlinearities as far as possible, and ii) to reduce the dimensionality of the system, if possible. Two rigorous mathematical techniques that allow substantial progress along both lines of approach are transformation into the normal form and the center manifold reduction. These are the most powerful analytical methods available in the local theory of dynamical systems and they form the fundamental step towards the bifurcation analysis, studies of chaotic motion, control theory etc. of parameter dependent dynamical systems.

Bifurcation theory is a subject with classical mathematical origin. Its modern development was initiated by Poincare in the development of the qualitative theory of differential equations. The word bifurcation meaning some sort of branching process in the qualitative, topological picture of object for the change of parameters on which the object depends. The object in question can be extremely diverse: for real or complex curves, or surfaces, manifolds, vector fields,

differential or integral equations. In recent years, this theory has undergone a tremendous development with an infusion of new ideas and methods from dynamical system theory, singularity theory, group theory and computer assisted studies of dynamics.

Nonlinear dynamics and chaotic and complex system constitute some of the fascinating development of the late twentieth century mathematics and physics. A century ago, researchers discovered that some apparently simple system can have very complicated motions. Not only their behavior extremely sensitive to the precise starting conditions, but they never settle to any reasonable final state with a recognizable fixed pattern. Although their movements look smooth over short span of time, they seem to jump unpredictably and indefinitely when their state of the motion are compared over large time intervals.

In Mathematics, chaos theory describes the behavior of certain dynamical systems, that may exhibit dynamics that are highly sensitive to initial conditions. As a result of this sensitivity to which manifest itself as an exponential growth of variation in the initial condition, the behavior of chaotic systems appears to be random. This happens even though these system are deterministic, meaning that their future dynamics are fully defined by their initial conditions, with no random element involved. This behavior is known as deterministic chaos or simply chaos.

Chaotic behavior has been observed in the laboratory in a variety of systems including fluid dynamics, electrical circuits, lasers, oscillating chemical reactions, mechanical and magneto-mechanical devices. Observations of chaotic phenomena in nature include the transport of fluid, the dynamics of satellite in solar system, the time evolution of the magnetic field of celestial bodies, population growth in ecology, the dynamics of action potential in neurons and molecular vibrations and so on. Our everyday experience on chaotic system are the weather and climate.

Chaos is not only here to stay, but will challenge many of our assumptions about the typical behavior of dynamical systems. Since mechanics underlies our view of nature, we will probably have to modify some of our ideas concerning the harmony and beauty of universe.

On the other hand, wavelet theory , developed mostly over the past 30 years, has generated tremendous amount ofInterest in in many areas of research both in pure and applied Mathematics, Physics, Computer science, and Engineering. Methematical methods based on Wavelet theory can be viewed as a method in which the approximating function is defined by use of a multiresolution technique based on scaling or wavelet functions, similar to those used in signal and image processing. Its desirable advantages are multi-resolution properties and various basis functions for structural analysis. By means of "two-scale relations" of scaling functions, the scale adopted can be changed freely according to requirements to improve analysis accuracy. So wavelet method is well argued by many researchers not only in numerical analysis domains but also in structural analysis fields. The continuous wavelet transform was discovered by Grossmann & Morlet (1984) and the orthogonal wavelet transform by Lemarie & Meyer (1986). Daubechies (1988) developed orthogonal bases made of compactly supported wavelets, and Mallat (1989) designed the first wavelet transform algorithm.

Wavelets are basic functions that are localized in both physical space (owing to their fast decay or even compact support) and wave-number space (owing to their vanishing moments and smoothness). For comparison, the classical Fourier transform is based on functions (sines and cosines) that are well localized in wave-number space, but do not provide localization in physical space as they have global support. Due to this space/scale localization, the wavelet transform provides both spatial and scale (frequency) information, whereas the Fourier transform only provides frequency information.

A function u(x) can be represented in terms of wavelet basis functions as

$$u(x) = \sum_{i \in I_{\varphi}^{0}} u_i{}^{0}\, \varphi^{0}{}_{i}(x) + \sum_{j=0}^{+\infty} \sum_{\mu=1}^{2^d-1} \sum_i u_i^{\mu.j}\, \psi_i^{\mu,j}(x), \qquad (1)$$

where u_i^0 and $\varphi_i^0(x)$ are the scaling coefficients and functions on the lowest level of resolution, and $u_i^{\mu.j}$ *and* $\psi_i^{\mu,j}(x)$ the wavelet coefficients and basic functions of different families, μ, and levels of resolution, j. Bold subscripts denote indices in d- dimensional space ; i.e., i= $(i_{1,\ldots\ldots,} i_d)$, $I_{\emptyset}^{0}$ and $I_i^{\mu,j}$ are d-dimensional index sets, associated with scaling functions at zero level of resolution and wavelets of family μ and level j. One may think of a wavelet decomposition as a multilevel or

multiresolution representation of a function, in which each level of resolution j (except the coarsest one) consists of wavelet ψ_i^j or a family of wavelets $\psi_i^{\mu,j}$ having the same scale but located at different positions. Scaling function coefficients, u_i^0, represent the averaged values of the field, whereas the wavelet coefficients, $u_i^{\mu,j}$, measure the fluctuations (details) of the field u(x) at j scale and around position of the wavelet $\psi_i^{\mu,j}(x)$. In d dimensions, there are 2^d-1 distinctive d-dimensional wavelet families (Daubechies 1992). The above wavelet decomposition can be also applied to vector fields component-wise.

Wavelet and scaling function coefficients can be obtained, respectively, by multiplying equation (1) by the bi-orthogonal dual wavelets $\psi_i^{\mu,j}(x)$ and scaling functions $\varphi_i^0(x)$ and integrating over the domain, resulting in the following definition of the cofficients:

$$u_i^j = \langle u(x), \varphi_i^0(x) \rangle,\ u_i^{\mu,j} = \langle u(x), \psi_i^{\mu,j}(x) \rangle, \qquad (2)$$

Where $\langle .,. \rangle$ denotes the L^2-inner product, defined by $\langle u, g \rangle = \int u(x)g(x)dx$. In the case of orthogonal wavelets, the dual wavelets are identical to primary ones, i.e., $\psi = \tilde{\psi}$. The wavelet decomposition is computationally efficient, thanks to the existence of fast recursive transform that allows computation of wavelet coefficients from the scaling coefficients at a finer level of resolution. The fast wavelet transform yields an efficient O(N) algorithm to compute the N wavelet coefficients $u_i^{\mu,j}$ from the N grid point values of the function u(x) (for a more general discussion on wavelets, see Daubechies 1992, Mallat 1999).

There are many heuristic techniques for obtaining exact solutions of differential equations. But each of them works only for a limited class of problems. How can one solve differential equations of an unfamiliar type? Surprisingly, most conventional techniques have an underlying root; symmetries of differential equations. It is often possible to find symmetries of given differential equation (even of complicated structure) and then use then systematically to obtain exact solutions. Symmetries can also be useful to simplify problems and to understand the comprehensive aspects like bifurcations of physical processes described by nonlinear differential equations.

More than a century ago the Norwegian Mathematician Marius Sophus Lie in his short life span put forward many of the fundamental ideas behind the symmetry based technique for the solution of differential equations. Most of this ideas are essentially the confluence of algebra, calculus and geometry, three pillars of Mathematics, are so far reaching that they are still basis of systematic analysis of mathematical models used to describe physical processes. These methods are algorithmic and provide the user with a powerful range of tools for studying new equations. It has been realized that no one who deals with mathematical modeling of physical processes through differential equations can afford to be ignorant of this promising method.

Recently, it has been observed that the finite difference and Ritz-type methods of the pre-computer era had largely been replaced in the computer era by finite element method, boundary element method. Meshless method, and in the near future it may be the turn for wavelet method for numerical solution of partial differential equations used for describing physical processes.

I, thus intend to carry my research work on the improvement of existing methods for analytical or numerical solutions of nonlinear ordinary or partial differential equations e.g., Zakharov-like equations in plasma dynamics, Navier-Stokes equations in fluid dynamics, which are used as models for mathematical analysis of natural phenomena of current interest.

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